Weinberg 2.1

$$p = \left(\frac{E}{c}, 0, |\vec{p}|, 0 \right)$$

$$L = \begin{pmatrix} \gamma & & \sqrt{\gamma^2-1} & \\ & 1 & & \\ \sqrt{\gamma^2-1} & & \gamma & \\ & & & 1 \end{pmatrix} = \begin{pmatrix} \gamma & & \gamma\beta & \\ & 1 & & \\ \gamma\beta & & \gamma & \\ & & & 1 \end{pmatrix}.$$

Λ: boost in z direction by $|\vec{v}|$.

$$\Lambda = \begin{pmatrix} \gamma' & & & \\ & 1 & & \\ & & 1 & -\gamma'|\vec{v}|/c \\ & & -\gamma'|\vec{v}|/c & \gamma' \end{pmatrix} \begin{pmatrix} E/c \\ 0 \\ |\vec{p}| \\ 0 \end{pmatrix}$$

$$\Lambda p = \begin{pmatrix} \gamma' \frac{E}{c} \\ 0 \\ |\vec{p}| \\ -\gamma' |\vec{v}|/c \, |\vec{p}| . \end{pmatrix}$$

$\Leftarrow P$

$$L^{-1}(\Lambda p) = L\left(\begin{bmatrix} \gamma' E/c \\ 0 \\ -|\vec{p}| \\ +\gamma' |\vec{v}|/c \, |\vec{p}| \end{bmatrix} \right)$$

which will be pure boost.

$$\mathbb{P} = \left[\gamma' E/c,\ 0,\ -|\vec{p}|,\ \gamma' \frac{|\vec{v}|}{c} |\vec{p}| \right]$$

$$\hat{\mathbb{P}} = \frac{\vec{\mathbb{P}}}{|\mathbb{P}|} = \frac{1}{\sqrt{|\vec{p}|^2 + \gamma'^2 \frac{|\vec{v}|^2}{c^2} |\vec{p}|^2}} \left[0,\ -|\vec{p}|,\ \gamma' \frac{|\vec{v}|}{c} |\vec{p}| \right]$$

$$\gamma'' = \sqrt{|\vec{p}|^2 + M^2} \,/\, M.$$

$$L^{-1}(\Lambda_p) = \begin{pmatrix} \gamma'' & 0 & \sqrt{\gamma''^2 - 1}\, \hat{\mathbb{P}}_y & \sqrt{\gamma''^2 - 1}\, \hat{\mathbb{P}}_z \\ 0 & 1 & 0 & 0 \\ \sqrt{\gamma''^2 - 1}\, \hat{\mathbb{P}}_y & 0 & 1 + (\gamma'' - 1) \hat{\mathbb{P}}_y^2 & 1 + (\gamma'' - 1) \hat{\mathbb{P}}_y \hat{\mathbb{P}}_z \\ \sqrt{\gamma''^2 - 1}\, \hat{\mathbb{P}}_z & 0 & 1 + (\gamma'' - 1) \hat{\mathbb{P}}_y \hat{\mathbb{P}}_z & 1 + (\gamma'' - 1) \hat{\mathbb{P}}_z^2 \end{pmatrix}$$

$L^{-1}(\Lambda_p) \Lambda L(p)$ = some rotation, we already know.

This rotation will take place in the y-z plane

The angle is given by Thomas Precession formula:

$$\Delta \Omega \approx (\gamma - 1) \frac{\beta_z}{\beta_y} \qquad \text{for } |\vec{v}| \text{ small}.$$

where $\beta_z = \frac{|\vec{v}|}{c}$, $\beta_y = \frac{|\vec{p}|}{\gamma M c}$, $\gamma = \sqrt{|\vec{p}|^2 + M^2} / M$.

We then will have $L^{-1}(\Lambda p)\Lambda L(p) \approx R_x(\Delta\Omega)$,

where $\Delta\Omega$ given above.

The state that the second observer observes will have the momentum transformed according to usual Lorentz transformation.

The spin that the second observer observes will not be σ in z. Rather, it will depend on the spin 1 representation of $SO(3)$.

Davidson Cheng

2.17.2024.